Minimal Length in Quantum Gravity, Equivalence Principle and Holographic Entropy Bound

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Abstract

A possible discrepancy has been found between the results of a neutron interferometry experiment and Quantum Mechanics. This experiment suggests that the weak equivalence principle is violated at small length scales, which quantum mechanics cannot explain. In this paper, we investigated whether the Generalized Uncertainty Principle (GUP), proposed by some approaches to quantum gravity such as String Theory and Doubly Special Relativity Theories (DSR), can explain the violation of the weak equivalence principle at small length scales. We also investigated the consequences of the GUP on the Liouville theorem in statistical mechanics. We have found a new form of invariant phase space in the presence of GUP. This result should modify the density states and affect the calculation of the entropy bound of local quantum field theory, the cosmological constant, black body radiation, etc. Furthermore, such modification may have observable consequences at length scales much larger than the Planck scale. This modification leads to a $\sqrt{A}$-type correction to the bound of the maximal entropy of a bosonic field which would definitely shed some light on the holographic theory.

1 Inroduction

The existence of a minimal length is one of the most interesting predictions of some approaches related to quantum gravity such as String Theory and Black hole physics. This is a consequence of Perturbation String Theory since strings can not interact at distances smaller than their size. One of the interesting phenomenological implications of the existence of the minimal measurable length is the modification of the standard commutation relation, between position and momentum, in usual quantum mechanics to the so-called generalized uncertainty principle (GUP). Recently, we proposed the GUP in [1,2] which is consistent with Doubly Special Relativity (DSR) theories, String Theory and Black Holes Physics *and* which ensures $[x_i, x_j] = 0 = [p_i, p_j]$ (via the Jacobi identity).

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$$[x_i, p_j] = i\hbar \left[\delta_{ij} - \alpha \left(p\delta_{ij} + \frac{p_i p_j}{p} \right) + \alpha^2 \left(p^2 \delta_{ij} + 3p_i p_j \right) \right] \tag{1.1}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 - 2\alpha <p> + 4\alpha^2 <p^2> \right]$$

$$\geq \frac{\hbar}{2} \left[1 + \left(\frac{\alpha}{\sqrt{\langle p^2 \rangle}} + 4\alpha^2 \right) \Delta p^2 + 4\alpha^2 \langle p \rangle^2 - 2\alpha \sqrt{\langle p^2 \rangle} \right] \tag{1.2}$$

where $\alpha = \alpha_0/M_{Pl}c = \alpha_0 \ell_{Pl}/\hbar$, $M_{Pl} = $ Planck mass, $\ell_{Pl} \approx 10^{-35}$ $m = $ Planck length, and $M_{Pl}c^2 = $ Planck energy $\approx 10^{19}$ GeV. Various versions of the GUP have been proposed by many authors, motivated by String Theory, Black Hole Physics, DSR etc, see e.g. [6–11], and for investigating phenomenological implications see [3–5].) Note that Eqs. (1.1) and (1.2) are approximately covariant under DSR transformations [11]. Since DSR transformations preserve both speed of light, and invariant energy scale, it is not surprising that Eqs. (1.1) and (1.2) imply the existence of minimum measurable length and maximum measurable momentum

$$\Delta x \geq (\Delta x)_{min} \approx \alpha_0 \ell_{Pl} \tag{1.3}$$

$$\Delta p \leq (\Delta p)_{max} \approx \frac{M_{Pl}c}{\alpha_0} . \tag{1.4}$$

It can be shown that the following definitions

$$x_i = x_{0i} , \quad p_i = p_{0i} \left(1 - \alpha p_0 + 2\alpha^2 p_0^2 \right) , \tag{1.5}$$

(with x_{0i}, p_{0j} satisfying the canonical commutation relations $[x_{0i}, p_{0j}] = i\hbar \, \delta_{ij}$, such that $p_{0i} = -i\hbar \partial/\partial x_{0i}$) satisfy Eq.(1.1). In [1], we have shown that any non-relativistic Hamiltonian of the form $H = p^2/2m + V(\vec{r})$ can be written as $H = p_0^2/2m - (\alpha/m)p_0^3 + + V(r) + \mathcal{O}(\alpha^2)$ using Eq.(1.5), where the second term can be treated as a perturbation. Now, the third order Schrödinger equation has a new *non-perturbative* solution of the form $\psi \sim e^{ix/2a\hbar}$. When applied to an elementary particle, it implies that the space which confines it must be discrete.

$$\frac{L}{a\hbar} = \frac{L}{a_0 \ell_{Pl}} = 2p\pi + \theta , \; p \in \mathbb{N} \tag{1.6}$$

This suggests that space itself is discrete, and that all measurable lengths are quantized in units of a fundamental minimum measurable length (which can be the Planck length).

Considering the relativistic case in [2] was important for many reasons. The relativistic particles are natural candidates for studying the nature of spacetime near the Planck scale. Also, most of the elementary particles in the nature are fermions, obeying some form of the Dirac equation. Furthermore, It is easier to investigate whether the discreteness of space exist in 2 and 3 dimensions by studying Dirac equation with GUP. We have shown that to confine the particle in the D-dimensional box, the dimensions of the box would have to be quantized in multiples of a fundamental length, which can be the Planck length.

The scope of the present work is to investigate the effect of Quantum Gravity Corrections on the equivalence principle and the holographic entropy bound. In section 2, we investigate Heisenberg equations of motion in the presence of the GUP, we found that the acceleration is no longer mass-independent because of the mass-dependence through the momentum p. Therefore, the equivalence principle is dynamically violated. In section 3, we tackle a naturally arising question of whether the number of states inside a volume of phase space does not change with time in the presence of the GUP. So, we calculate the consequences of the GUP on the Liouville theorem in statistical mechanics. We applied our approach on the entropy bound of local quantum field theory. This leads to a $\sqrt{A}$-type correction to the bound of the maximal entropy of a quantum field.

2   The Equivalence principle at short distance

Quantum Mechanics does not violate Equivalence principle. This can be shown from studying Heisenberg equations of motion. For simplicity, consider $1-dimensional$ motion with the Hamiltonian given by

$$H = \frac{P^2}{2m} + V(x). \tag{2.1}$$

The Heisenberg equations of motion read,

$$\dot{x} = \frac{1}{i\hbar}[x, H] = \frac{p}{m}, \tag{2.2}$$

$$\dot{p} = \frac{1}{i\hbar}[p, H] = -\frac{\partial V}{\partial x}. \tag{2.3}$$

These equations ensure that the momentum at the quantum level is $p = m\dot{x}$ and the acceleration $\ddot{x}$ is mass-independent like in classical physics. It is obvious that the equivalence principle is preserved at the quantum level, and it is clear that this result possibly contradicts experimental results [12].

Let us study Eq(1.1) at the classical limit using the correspondence between commutator in quantum mechanics and poisson bracket in classical mechanics,

$$\frac{1}{i\hbar}[\hat{P}, \hat{Q}] \Longrightarrow \{P, Q\}, \tag{2.4}$$

so the classical limit of Eq(1.1) give

$$\{x_i, p_j\} = \delta_{ij} - \alpha(p\delta_{ij} + \frac{p_i p_j}{p}) + \alpha^2(p^2\delta_{ij} + 3p_i p_j). \tag{2.5}$$

The equations of motion are given by

$$\dot{x}_i = \{x_i, H\} = \{x_i, p_j\}\frac{\partial H}{\partial p_j},$$
$$\dot{p}_i = \{p_i, H\} = -\{x_j, p_i\}\frac{\partial H}{\partial x_j}. \tag{2.6}$$

3

Consider the effect of the GUP on $1 - dimensional$ motion with the Hamiltonian given by,

$$H = \frac{P^2}{2m} + V(x). \tag{2.7}$$

The equations of motion will be modified as follows,

$$\dot{x} = \{x, H\} = (1 - 2\alpha p)\frac{p}{m}, \tag{2.8}$$

$$\dot{p} = \{p, H\} = (1 - 2\alpha p)(-\frac{\partial V}{\partial x}), \tag{2.9}$$

where the momentum p is no longer equal to $m\dot{x}$.

Using (2.8,2.9), we can derive the acceleration given by,

$$\ddot{x} = -(1 - 6\alpha p)\frac{\partial V}{\partial x}. \tag{2.10}$$

Notice that if the force $F = -\frac{\partial V}{\partial x}$ is gravitational and proportional to the mass m, the acceleration $\ddot{x}$ is not mass-independent because of the mass-dependence through the momentum p. Therefore, the equivalence principle is dynamically violated because of generalized uncertainty principle. Since the GUP is an aspect of various approaches to Quantum Gravity such as String Theory and Doubly Special Relativity (or DSR) Theories, as well as black hole physics, it is promising to predict the upper bounds on the quantum gravity parameter compatible with the experiment that was done in [12]. This result agrees, too, with cosmological implications of the dark sector where a long-range force acting only between nonbaryonic particles would be associated with a large violation of the weak equivalence principle [13]. The violation of equivalence principle has been obtained, too, in the context of string theory [14] where the extended nature of strings are subject to tidal forces and do not follow geodesics.

3   The GUP and Liouville theorem

In this section, we continue our investigation of the consequences of our proposed commutation relation of Eq(1.1). What we are looking for is an analog of the Liouville theorem in presence of the GUP. We should make sure that the number of states inside volume of phase space does not change with time revolution in presence of the GUP. If this is the case, this should modify the density states and affect the Entropy bound of local quantum field theory, the Cosmological constant, black body radiation, etc. Furthermore, such modification may have observable consequences at length scales much larger than the Planck scale. The Liouville theorem has been studied before with different versions of GUP, see e.g. [15].

Since we are seeking the number of states inside a volume of phase space that does not change with time, we assume the time evolutions of the position and momentum during δt are

$$x_i' = x_i + \delta x_i ,$$

$$p'_i = p_i + \delta p_i \,, \tag{3.1}$$

where

$$\delta x_i = \{x_i, p_j\} \frac{\partial H}{\partial p_j} \delta t \,,$$
$$\delta p_i = -\{x_j, p_i\} \frac{\partial H}{\partial x_j} \delta t \,. \tag{3.2}$$

The infinitesimal phase space volume after this infinitesimal time evolution is

$$d^D\mathbf{x}' \, d^D\mathbf{p}' = \left| \frac{\partial(x'_1, \cdots, x'_D, p'_1, \cdots, p'_D)}{\partial(x_1, \cdots, x_D, p_1, \cdots, p_D)} \right| d^D\mathbf{x} \, d^D\mathbf{p} \,. \tag{3.3}$$

Using Eq(3.1), the Jacobian reads to the first order in δt

$$\left| \frac{\partial(x'_1, \cdots, x'_D, p'_1, \cdots, p'_D)}{\partial(x_1, \cdots, x_D, p_1, \cdots, p_D)} \right| = 1 + \left(\frac{\partial \delta x_i}{\partial x_i} + \frac{\partial \delta p_i}{\partial p_i} \right) + \cdots \,. \tag{3.4}$$

Using Eq(3.2), we get

$$
\begin{aligned}
\left(\frac{\partial \delta x_i}{\partial x_i} + \frac{\partial \delta p_i}{\partial p_i} \right) &\frac{1}{\delta t} \\
&= \frac{\partial}{\partial x_i} \left[\{x_i, p_j\} \frac{\partial H}{\partial p_j} \right] - \frac{\partial}{\partial p_i} \left[\{x_j, p_i\} \frac{\partial H}{\partial x_j} \right] \\
&= \left[\frac{\partial}{\partial x_i} \{x_i, p_j\} \right] \frac{\partial H}{\partial p_j} + \{x_i, p_j\} \frac{\partial^2 H}{\partial x_i \partial p_j} \frac{\partial H}{\partial x_j} \\
&\quad - \left[\frac{\partial}{\partial p_i} \{x_j, p_i\} \right] \frac{\partial H}{\partial x_j} - \{x_j, p_i\} \frac{\partial^2 H}{\partial p_j \partial x_i} \\
&= - \left[\frac{\partial}{\partial p_i} \{x_j, p_i\} \right] \frac{\partial H}{\partial x_j} \\
&= -\frac{\partial}{\partial p_i} \left[\delta_{ij} - \alpha(p\delta_{ij} + \frac{p_i p_j}{p}) \right] \frac{\partial H}{\partial x_j} \\
&= \frac{\partial}{\partial p_i} \alpha(p\delta_{ij} + \frac{p_i p_j}{p}) \frac{\partial H}{\partial x_j} \\
&= \alpha(D+1) \frac{p_j}{p} \frac{\partial H}{\partial x_j}. \tag{3.5}
\end{aligned}
$$

The infinitesimal phase space volume after this infinitesimal evolution up to first order in α and δt is

$$d^D\mathbf{x}' \, d^D\mathbf{p}' = d^D\mathbf{x} \, d^D\mathbf{p} \left[1 + \alpha(D+1) \frac{p_i}{p} \frac{\partial H}{\partial x_i} \delta t \right]. \tag{3.6}$$

Now we are seeking the analog of the Liouville theorem in which the weighted phase space volume is invariant under time evolution. Let us check the infinitesimal evolution of $(1 - \alpha p')$ up to first order in α and δt

$$
\begin{aligned}
(1 - \alpha p') &= 1 - \alpha \sqrt{p_i' p_i'} \\
&= 1 - \alpha \left[(p_i + \delta p_i)(p_i + \delta p_i) \right]^{\frac{1}{2}} \\
&\approx 1 - \alpha (p^2 + 2 p_i \delta p_i)^{\frac{1}{2}} \\
&\approx 1 - \alpha \left[p^2 - 2 p_i \{x_i, p_j\} \frac{\partial H}{\partial x_j} \delta t \right]^{\frac{1}{2}} \\
&\approx 1 - \alpha \left[p - \frac{1}{p}(p_j - 2\alpha p p_j) \frac{\partial H}{\partial x_j} \delta t \right] \\
&\approx (1 - \alpha p) + \alpha \frac{p_j}{p}(1 - 2\alpha p) \frac{\partial H}{\partial x_j} \delta t \\
&\approx (1 - \alpha p) \left[1 + \alpha \frac{p_j}{p} \frac{1 - 2\alpha p}{1 - \alpha p} \frac{\partial H}{\partial x_j} \delta t \right] \\
&\approx (1 - \alpha p) \left[1 + \alpha \frac{p_j}{p} \frac{\partial H}{\partial x_j} \delta t \right].
\end{aligned}
\tag{3.7}
$$

Therefore, we get to first order in α and δt,

$$
(1 - \alpha p')^{-D-1} = (1 - \alpha p)^{-D-1} \left[1 - (D+1)\alpha \frac{p_j}{p} \frac{\partial H}{\partial x_j} \delta t \right]
\tag{3.8}
$$

This result in the following expression is invariant under time evolution!

$$
\frac{d^D \mathbf{x}' \, d^D \mathbf{p}'}{(1 - \alpha p')^{D+1}} = \frac{d^D \mathbf{x} \, d^D \mathbf{p}}{(1 - \alpha p)^{D+1}} .
\tag{3.9}
$$

If we integrate over the coordinates, the invariant phase space volume of Eq. (3.9) will be

$$
\frac{V \, d^D \mathbf{p}}{(1 - \alpha p)^{D+1}} ,
\tag{3.10}
$$

Where V is the coordinate space volume. The number of quantum states per momentum space volume can be assumed to be

$$
\frac{V}{(2\pi\hbar)^D} \frac{d^D \mathbf{p}}{(1 - \alpha p)^{D+1}} .
\tag{3.11}
$$

The modification in the number of quantum states per momentum space volume in (3.11) should have consequences on the calculation of the entropy bound of local quantum field theory, the cosmological constant, black body radiation, etc. In this paper, we are investigating its consequences on the entropy bound of local quantum field. In the following two subsections we briefly introduce the holographic entropy bound proposed by't Hooft [16] and entropy bound of Local quantum field proposed by Yurtsever and Aste [17,18,20]. In subsection(3.2) we treat the effects of the GUP on the entropy bound of Local quantum field.

6

3.1 The Holographic Entropy Bound and Local Quantum Field Theory

The entropy of a closed spacelike surface containing quantum bosonic field has been studied by 't Hooft [16] . For the field states to be observable for outside world, 't Hooft assumed that their energy inside the surface should be less than 1/4 times its linear dimensions, otherwise the surface would lie within the Schwarzschild radius [16].

If the bosonic quantum fields are confined to closed spacelike surface at a temperature T, the energy of the most probable state is

$$E = a_1 Z T^4 V, \tag{3.12}$$

where Z is the number of different fundamental particle types with mass less than T and a_1 a numerical constant of order one, all in natural units.

Now turning to the total entropy S, it is found that it is given by

$$S = a_2 Z V T^3, \tag{3.13}$$

where a_2 is another numeric constant of order one.

The Schwarzschild limit requires that

$$2E < \frac{V}{\frac{4}{3}\pi}. \tag{3.14}$$

Using Eq(3.12), one finds

$$T < a_3 Z^{-\frac{1}{4}} V^{-\frac{1}{6}}, \tag{3.15}$$

so the entropy bound is given by

$$S < a_4 Z^{\frac{1}{4}} V^{\frac{1}{2}} = a_4 Z^{\frac{1}{4}} A^{\frac{3}{4}}, \tag{3.16}$$

where A is the boundary area of the system. At low temperatures, Z is limited by a dimensionless number , then this entropy is small compared to that of a black hole, if the area A is sufficiently large. The black hole is the limit of maximum entropy

$$S_{max} = \frac{1}{4} A. \tag{3.17}$$

Therefore, for any closed surface without worrying about its geometry inside, all physics can be represented by degrees of freedom on this surface itself. This implies that the quantum gravity can be described by a topological quantum field theory, for which all physical degrees of freedom can be projected onto the boundary [16]. This is know as Holographic Principle.

According to Yurtsevers paper [17], the holographic entropy bound can be derived from elementary flat-spacetime quantum field theory when the total energy of Fock states is in a stable configuration against gravitational collapse by imposing a cutoff on the maximum energy of the field modes of the order of the Planck energy. This leads to an entropy bound of holographic type.

Consider a massless bosonic field confined to cubic box of size L , as has been done in [17–21], the total number of the quantized modes is given by

$$N = \sum_{\vec{k}} 1 \rightarrow \frac{L^3}{(2\pi)^3} \int d^3\vec{p} = \frac{L^3}{2\pi^2} \int_0^\Lambda p^2 dp = \frac{\Lambda^3 L^3}{6\pi^2}, \tag{3.18}$$

where Λ is the UV energy cutoff of the LQFT. The UV cutoff makes N finite. The Fock states can be constructed by assigning occupying number n_i to these N different modes

$$\mid \Psi >= \mid n(\vec{k}_1), n(\vec{k}_2), \cdots, n(\vec{k}_N) > \quad \rightarrow \quad \mid n_1, n_2, \cdots, n_N >, \tag{3.19}$$

The dimension of the Hilbert space is calculated by the number of occupancies $\{n_i\}$ which is finite if it is bounded. The non-gravitational collapse condition leads to finiteness of the Hilbert space.

$$E = \sum_{i=1}^N n_i \omega_i \leq E_{BH} = L. \tag{3.20}$$

It can be observed that N particle state with one particle occupying one mode ($n_i = 1$) corresponds to the lowest energy state with N modes simultaneously excited. In this case, it should satisfy the gravitational stability condition of Eq.(3.20). Hence, the energy bound is given by

$$E \rightarrow \frac{L^3}{2\pi^2} \int_0^\Lambda p^3 dp = \frac{\Lambda^4 L^3}{8\pi^2} \leq E_{BH}. \tag{3.21}$$

The last inequality implies

$$\Lambda^2 \leq \frac{1}{L}. \tag{3.22}$$

The maximum entropy is given by

$$S_{\max} = -\sum_{j=1}^W \frac{1}{W} \ln \frac{1}{W} = \ln W, \tag{3.23}$$

where the bound of W is determined by

$$W = \dim\mathcal{H} < \sum_{m=0}^N \frac{z^m}{(m!)^2} \leq \sum_{m=0}^\infty \frac{z^m}{(m!)^2} = I_0(2\sqrt{z}) \sim \frac{e^{2\sqrt{z}}}{\sqrt{4\pi\sqrt{z}}}. \tag{3.24}$$

Here I_0 is the zeroth-order Bessel function of the second kind. Since z is given by

$$z = \sum_{i=1}^N L_i \rightarrow \frac{L^3}{2\pi^2} \int_0^\Lambda \left[\frac{E_{BH}}{p} \right] p^2 dp = \frac{\Lambda^2 L^4}{4\pi^2}. \tag{3.25}$$

Using UV-IR relation of Eq(3.22), the bound can be given as follows

$$z \leq L^3. \tag{3.26}$$

Since the boundary area of the system is given by

$$A \sim L^2, \tag{3.27}$$

therefore, the bound for the maximum entropy of Eq(3.23) will be given by

$$S_{\max} = \ln W \leq A^{3/4}. \tag{3.28}$$

This is just a brief summary of determining entropy bound by using the Local Quantum Field Theory (LQFT).

3.2 The effect of GUP on The Holographic Entropy Bound and LQFT

Consider a massless bosonic field confined to cubic box of size L , as has been done in subsection 3.1, but now with including the GUP modification. Using Eq(3.11), the total number of the quantized modes will be modified as follows,

$$N \to \frac{L^3}{2\pi^2} \int_0^\Lambda \frac{p^2 dp}{(1 - \alpha p)^4} \approx \frac{L^3}{2\pi^2} \left(\frac{\Lambda^3}{3} + \alpha \Lambda^4 \right). \tag{3.29}$$

We note the total number of states is increased due to GUP correction. Note that, this result is valid subject to:

$$\frac{1}{\alpha} > \Lambda, \tag{3.30}$$

otherwise, the number of states will be infinite or negative number. This means α gives a boundary on the cutoff Λ.

Now turning to the modifications implied by GUP on the energy bound up to the first order of α , we find

$$E \to \frac{L^3}{2\pi^2} \int_0^\Lambda \frac{p^3 dp}{(1 - \alpha p)^4} \approx \frac{L^3}{2\pi^2} \left(\frac{\Lambda^4}{4} + \alpha \frac{4\Lambda^5}{5} \right) \leq E_{BH}. \tag{3.31}$$

Using equations (3.20,3.22) with the last inequality (3.31), we get the following UV-IR relation up to the first order of α

$$\frac{L^3}{8\pi^2} \left(\Lambda^4 + \alpha \frac{16\Lambda^5}{5} \right) \leq L,$$
$$\Lambda^4 \left(1 + \alpha \frac{16\Lambda}{5} \right) \leq \frac{1}{L^2},$$
$$\Lambda^2 \leq \frac{1}{L} \left(1 - \frac{8\alpha}{5L^{\frac{1}{2}}} \right). \tag{3.32}$$

On the other side, the modified maximum entropy has been calculated according to the following procedure

$$S_{\max} = \ln W, \tag{3.33}$$

with $W \sim e^{2\sqrt{z}}$. Since z is given up to the first order of α by

$$z \to \frac{L^3}{2\pi^2} \int_0^\Lambda \left[\frac{E_{BH}}{p} \right] \frac{p^2 dp}{(1 - \alpha p)^4} \approx \frac{L^4}{2\pi^2} \left(\frac{\Lambda^2}{2} + \alpha \frac{4\Lambda^3}{3} \right), \tag{3.34}$$

one finds the bound when using UV-IR relation in Eq.(3.32)

$$z \leq \frac{L^4}{4\pi^2} \left(\Lambda^2 + \alpha \frac{8\Lambda^3}{3} \right),$$
$$z \leq \frac{L^4}{4\pi^2} \left(\frac{1}{L} \left(1 - \frac{8\alpha}{5L^{\frac{1}{2}}} \right) + \alpha \frac{8}{3L^{\frac{3}{2}}} \right),$$
$$z \leq L^3 + \frac{16\alpha L^{\frac{5}{2}}}{15}. \tag{3.35}$$

Using the boundary area of the system of Eq(3.27), we find the bound for the maximum entropy will be modified as follows,

$$S_{\max} = \ln W \leq A^{3/4} + \frac{16\alpha}{30} A^{1/2}, \tag{3.36}$$

which shows clearly that the upper bound is increased due to the GUP. This means that the the maximum entropy that can be stored in a bounded region of space has been increased due to the presence of the GUP or, in other words, by considering the minimal length in Quantum Gravity. This shows that the conjectured entropy of the truncated Fock space corrected by the GUP disagrees with 't Hooft's classical result which requires disagreement between the micro-canonical and canonical ensembles for a system with a large number of degrees of freedom due to the GUP-correction term. Then the holographic theory doesn't retain its good features. On the other side, since the GUP implies discreteness of space by itself as proposed in [1–3], therefore the discreteness of space will not leave the continuous symmetries such as rotation and Lorentz symmetry intact, which means by other words the holographic theory doesn't retain its good features [22]. Possibilities of violating Holographic theory near the Planck scale have been discussed by many authors see e.g [23]. This seems that holographic theory does not retain its good features by considering minimal length in Quantum Gravity.

4 Conclusions

In this paper we tackle the problem of studying the possible discrepancy that has been found between the results of neutron interferometry experiment and Quantum Mechanics. We investigated whether the GUP can explain the violation of weak equivalence principle at small length scales. We have shown that, by studying Heisenberg equations of motions in the presence of GUP, the acceleration is no longer mass-independent because of the mass-dependence through the momentum p. Therefore, the equivalence principle is dynamically violated.

We also investigated the consequences of the GUP on the Liouville theorem. We found a new form of an invariant phase space in the presence of the GUP. In the future, it would be nice to apply our approach on the calculations of the cosmological constant, black body radiation, etc. We applied our approach on the calculation of the entropy bound of local quantum field theory. This led to a $\sqrt{A}$-type correction to the bound of the maximal entropy of a bosonic field. This showed that the conjectured entropy of the truncated Fock space corrected by GUP disagrees with 't Hooft's classical result. This agreed with the discreteness of space implications which does not leave the continuous symmetries such as translation, rotation and full Lorentz symmetry intact, and hence the holographic theory doesn't retain its good features due to discreteness of space.

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